# Mathematics BSc, final exam

**1. Fundamental Concepts of Mathematics**

Basic concepts in set theory and logic: sets, propositions, operations and their properties. Cardinality of sets. Predicates and quantifiers. Arguments and proof or validation of an argument. Relations, equivalence relations, their relation to partitions, partial orders. Functions, injective, surjective and bijective functions, and their characterization. Composition of functions, inverse function.

**2. Number Theory I.**

Divisibility, greatest common divisor and least common multiple in the ring of integers. Euclidean division and its algorithm. Prime numbers, the fundamental theorem of arithmetic.

**3. Number Theory II.**

Congruences, residue classes. Euler's theorem, Fermat's theorem, Wilson's theorem. Linear congruences. Linear diophantine equations. Arithmetic functions.

**4. Linear Algebra**

The concept of a vector space over a field, subspace, linear independence and dependence, spanning set, basis, dimension. Matrices, matrix operations. Determinants and their properties. System of linear equations. Cramer's rule. Linear maps.

**5. Algebraic Structures I.**

Semigroups and groups. Subgroups, homomorphisms. Cosets, Lagrange's theorem, first isomorphism theorem. Permutation groups.

**6. Algebraic Structures II.**

Rings and fields. Subrings, subfields, homomorphisms, characteristic of a field. Ideals. Rings of polynomials, polynomial remainder theorem, irreducible polynomials. The fundamental theorem of algebra. Real and complex polynomials, equations.

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**7. Numerical Sequences and Series**

Sequences: bounded, monotone, convergent sequences and theorems on their relations.

Series: convergence, absolute convergence and their relation. Positive series, alternating series. Convergence criteria. Prominent sequences and series. Cauchy product of two series.

Theorems on the convergence of the Cauchy product.

**8. Power Series, Limit of Functions, Continuity.**

Domain of convergence of power series and limit function. Theorems on the continuity, differentiability and integralability of the limit function. Definition of elementary functions: exp, sin, cos, sh, ch functions and their properties. Graphs of the restrictions to real numbers. Limit of real valued functions with real variable. Limit of the sum, difference, product, ratio and composition of two functions. Limits of special functions. Methods for determining limits. Continuous functions and their properties.

**9. Differential Calculus**

Differentiability of real valued functions defined on reals: difference quotient, derivative, geometric interpretation. Connection between differentiability and continuity. Differentiability of sum, product, quotient and composition of functions and the inverse of a function. The L'Hospital's rule. Study of monotonicity, extreme values and convexity using differentiation. Analysis of functions.

**10. Theory of Integration I. Differential Equations**

Primitive functions. Operations with primitive functions. Classes of functions possessing primitive functions. Integration by parts. Integration by substitution. Separable differential equations. Solution of first order linear differential equations. Solution of second order linear differential equations.

**11. Theory of Integration II, Applications, Connection to Geometry**

The idea of the definite integral. Class of integrable functions, integral function. Newton-Leibniz theorem. Area of a plain region, arc length, volume of a body of revolution and area of a surface of revolution. Elementary notion of area, circumference, volume and surface.

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**12. Functions of Several Variables**

Limit and differentiability of functions of several variables. Differential matrix. Partial derivatives, directional derivative. Sufficient and necessary conditions of differentiability. Finding the extreme points of real valued functions in several variables. Line integral and its independence of the path. Definition of multiple integrals and its evaluation.

**13. Elementary Planar and Spatial Geometry, Constructions**

Spatial objects, positions, distances, angles. Triangles, quadrangles, polygons, polyhedra, convex geometry. Euclidean constructions. Prominent construction problems.

**14. Geometric Transformations**

Isometries, similarity transformations, affine and projective transformations and their properties. Invariants. Groups of transformations. Inversion and its applications.

**15. Fundamentals of Geometry**

The axioms of the euclidean plane geometry (only a sketch). Parallel postulate. Elements of spherical geometry, spherical trigonometry. Elements of real projective geometry. Homogeneous coordinates, cross ratio. Projectivities. collineations. Desargues's theorem, Pappus's hexagon theorem. Quadratic curves. Pascal's theorem and Brianchon's theorem.

**16. Analytic Geometry, Differential Geometry of Curves**

Vectors, operations, coordinates. Dot and cross products, their geometric applications. Metric problems. Description of point sets with equations (line, circle, conics. Geometry of circles. Parameterizations of curves. Basic concepts in differential geometry of curves. Frenet formulas.

**17. Combinatorics**

Permutations, variations, combinations (without repetition and with repetition). Binomial coefficients and their properties. Binomial theorem, principle of inclusion-exclusion. Fibonacci numbers, Catalan numbers. Basic concepts in graph theory, trees, Eulerian path, Hamiltonian cycle.

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**18. Probability Theory**

Combinatorial probability. Sample space, event algebra and its properties. Frequency and relative frequency. Axioms of probability. Classical probability space. Geometric probability. Conditional probability. Random variable, expectation, variance. Common discrete and continuous distributions.

**19. Number Theory III.**

Quadratic and higher order congruences. Legendre symbol, primitive roots, discrete logarithm. Estimations for $\pi(x)$ and for the n-th prime number. Sum of the reciprocals of the primes.

**20. Complex Analysis**

Holomorphic functions. Cauchy-Riemann equations. Complex integral. Cauchy's integral formula. Power series expansion of holomorphic functions. Laurent series.

**21. Real Analysis**

Introduction of Lebesgue integral. Lebesgue theorem. $L^2$ space, inner product. The trigonometric system. Trigonometric Fourier series. Dirichlet kernel. Dini criterion. Fejér's theorem.

**22. Operation Research**

Linear programming problem. Simplex algorithm. Duality. Duality theorems. Transportation problem.

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